The reluctant celebrity

A reclusive Russian claims to have solved a century-old mathematical problem — but his enigmatic personality is adding a fresh dimension to the proof-checking process. Emily Singer reports.

Proving the Poincaré conjecture will offer fresh insight into three-dimensional geometry.

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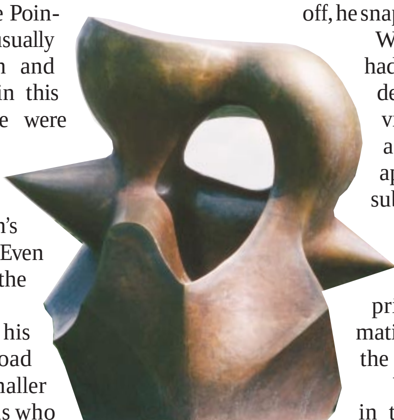
It takes guts for a young Russian to stride into the heart of the US mathematical establishment after claiming to have solved a problem that has baffled the discipline's finest minds for decades. But by the time that Grigory 'Grisha' Perelman, a 30-something recluse, arrived to talk at the Massachusetts Institute of Technology (MIT) in April 2003, it was clear that there was some substance behind his bravura.

Five months before, in a posting on the Internet[1], Perelman claimed to have proved the geometrization conjecture, a theory crucial for understanding three-dimensional surfaces. This would automatically prove the more famous Poincaré conjecture, which has stumped mathematicians for 100 years[2]. The world's top mathematicians had tried to pick holes in his argument — and although their job was far from complete, they already suspected that Perelman was onto something big.

"Every few years, someone claims to have solved the Poincaré conjecture. It is usually a chore to go through and find the problem. But in this case it was clear there were new and brilliant ideas," says Tom Mrowka, a mathematician at MIT who has studied Perelman's work. "It was so original. Even if there was a gap near the end, it was a big deal."

When Perelman gave his talks at MIT, both to a broad audience and to a smaller group of mathematicians who had spent several months studying his work, he had answers for every question that came up. "It was clear he had thought about all these issues before," says Mrowka. "He'd either point out that it was a trivial question or he'd have an answer."



M. VAUTIER/THE ART ARCHIVE

Mathematically this Henry Moore sculpture is the same as a doughnut.

The media soon got wind of the potential significance of Perelman's achievement. An article about his work appeared in *The New York Times* in mid-April, and when Perelman lectured at the Courant Institute of Mathematical Sciences in New York two weeks later, reporters mingled with mathematicians in the audience. This time, Perelman was much less effusive. He refused to answer reporters' questions or speculate about the implications of his work. When a photographer's flash went off, he snapped: "Don't do that!"

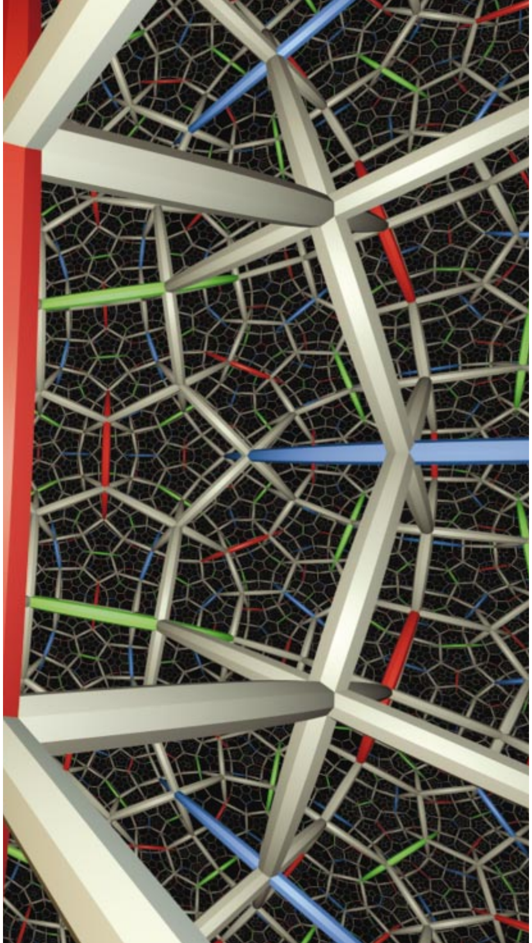
Within weeks Perelman had returned to Russia, evidently annoyed at the uninvited publicity. He ignored a flurry of job offers and apparently has no plans to submit his work to a peer-reviewed journal. He has also shown no interest in a US$1-million prize that awaits the mathematician who finally proves the Poincaré conjecture.

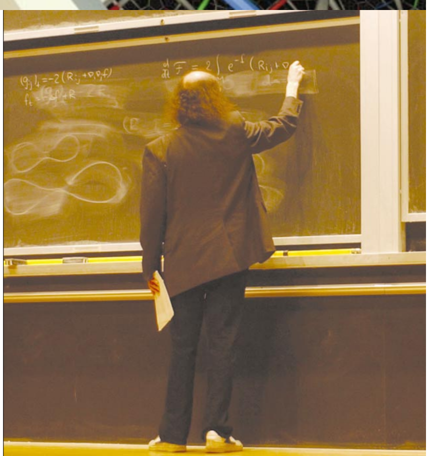
Why did Perelman react in this way to the acclaim generated by his work? It's hard to say. He talks about little other than mathematics, even with those who count themselves as his friends. Colleagues have only theories for his reclusive behaviour. And e-mails from *Nature*, requesting an interview for this article, went unanswered.

Scratching the surface

Perelman's work focuses on the geometrical properties of three-dimensional surfaces. In mathematical terms, the thin film that makes up a soap bubble is a two-dimensional surface that curves round to enclose a three-dimensional space. Similarly, there is a three-dimensional equivalent of the bubble's surface, called a three-sphere. Understanding such shapes could help mathematicians solve a host of topological problems, and perhaps even describe the shape of the Universe.

Nineteenth-century mathematicians had shown that any closed two-dimensional surface can be described — at a fundamental level — as one of two basic shapes: a sphere or a 'doughnut' with one or more holes. An egg is, in essence, the same as a smooth sphere; a coffee mug is a doughnut. Mathematically, an important characteristic separates spheres from doughnuts. A loop stretched around a sphere can always be shrunk down to a point — just as an elastic band round an egg can be pulled tight to a single point without losing contact with the egg's surface. This is not possible for a doughnut — a loop passing through the doughnut's hole cannot be pulled to a point without cutting through the doughnut





C. CHANG/THE DAILY PRINCETONIAN

Grisha Perelman presents his proof, which covers Poincaré's conjecture, in a lecture last year.

itself. In 1904, the French mathematician Henri Poincaré argued that a three-sphere should follow the rule for a two-dimensional sphere — but he was unable to prove it.

The geometrization conjecture is a more general statement about three-dimensional 'surfaces', derived in the late 1970s by William Thurston, now at Cornell University in Ithaca, New York. According to Thurston, all three-dimensional surfaces are made from eight basic geometries, and he theorized that every shape can be described using these building blocks. Poincaré's unsolved conjecture is a limited case of Thurston's theory.

Richard Hamilton, a mathematician at Columbia University in New York, had previously set out to prove Thurston's conjecture using the Ricci flow, a systematic procedure that smooths an object's surface into a simpler — homogenous — shape by spreading its curvature. But this isn't always easy. Some parts of the surface may transform faster than others, resulting in a 'lumpy' shape. These problem points, called singularities, prevented Hamilton from succeeding.

"You need to control the way singularities form," explains Jim Carlson, president of the Clay Mathematics Institute in Cambridge, Massachusetts. "Perelman found new inequalities that allow you to do that."

If it is correct, Perelman's work will have provided a proof for both the Poincaré and the geometrization conjecture, says Carlson. "If not, it develops tools and ideas that will bring the geometrization conjecture into reach," he adds. "There is a tremendous amount of excitement."

A flying start

Perelman first made an impression in the United States more than a decade ago. The young Russian spent two years of his early career working at the Courant Institute, the State University of New York at Stony Brook and the University of California, Berkeley. "He was already considered extremely brilliant; this was apparent in conversation and on the basis of his work," says Jeff Cheeger, a mathematician at the Courant Institute. But Perelman had an unusual reputation, even then. "He had long hair and long fingernails, several inches long," remarks one colleague. "When someone asked him why, he said it was so he could open a book at the exact page he wanted."

Perelman's early work was impressive enough to garner several job offers from US universities. But he turned them down, returning to Russia and a research position with the Steklov Institute of Mathematics in St Petersburg. At that point, Perelman effectively disappeared — he stopped publishing papers or discussing his research with colleagues. "We would occasionally ask where he was," says a friend. "No one seemed to know what he was doing." Even people at the Steklov Institute didn't know what he was working on.

But there were hints that he hadn't gone off the mathematical rails. "I was in touch with him a little bit," says Cheeger. "Enough to see he was following some developments closely." But no one knew whether he was working on something brilliant, or if he had just burned himself out.

In 2002, Perelman revealed what he had been working on for eight years. In November, he posted a paper[1] on an Internet preprint server outlining a proof of the geometrization conjecture. He also sent e-mails to a few mathematicians, telling them that they might be interested in the manuscript.

"I took a look and found it very interesting. It was a very important paper, so I decided to invite him here," says Gang Tian, a mathematician at MIT. At the same time, Perelman was invited to speak at Stony Brook. In the interim, Perelman posted a second paper[3] detailing technical arguments for his proof.

Tangible benefits of this work are hard for non-mathematicians to grasp. So attention has focused on the Poincaré conjecture and speculation about the Clay prize — a million-dollar cash award that the Clay Institute offers to anyone who can solve one of the seven toughest problems in mathematics.

Maths over money

Perelman has refused to discuss the Clay prize with the media and doesn't talk about it with friends. "He never mentioned the prize," says Tian, who played host to the Russian when he visited MIT. "He was only interested in talking about mathematics."

To win the prize, Perelman's work must survive two years of scrutiny by his peers. But even then, it is not clear whether Perelman would accept the award. He refused an earlier prize granted by the European Congress of Mathematics for work he did in the early 1990s. His reasons for doing so were unclear.

What is clear is that Perelman does not care much for money. "We tried to take him to a nice restaurant in Boston," recalls Mrowka. "I think he'd rather have had his bortsch. In some senses he is refreshing because he's totally committed to mathematics."

Perelman shows similar ambivalence towards the many job offers he received after his US lecture tour. Although universities are reluctant to give details, they say that they never heard back about their offers. At a private dinner during his stay at MIT, several people tried to convince Perelman to work in the United States, but he insisted that he could not be tempted.

Tian admits that he doesn't understand why. "Maybe he doesn't want his peaceful life disturbed; maybe he is right, once he has proved he has a great theorem, that is much more valuable," he says.

For now, mathematicians seem content to study Perelman's work without him. Workshops have sprung up throughout the United States, and the Clay Institute has planned conferences on the topic. Tian has gone through a large part of the second paper and says that everything looks correct so far. He hopes to complete his examination by the summer.

The two years of community scrutiny will be up in 2005, and then we'll know whether Perelman's peers deem his work worthy of a Clay prize. But given his track record, Perelman is unlikely to pay the award much heed. ■

Emily Singer recently completed a short internship in *Nature*'s Washington DC office.

1. Perelman, G. preprint at <http://www.arxiv.org/abs/math.DG/0211159> (2002).
2. Stewart, I. *Nature* **423,** 124–127 (2003).
3. Perelman, G. preprint at <http://www.arxiv.org/abs/math.DG/0303109> (2003).